Relative Music Theory

Writing the relationships between notes

Cyril Monkewitz | Revised edition · 22 November 2026

Open the online article to play the examples. This reading edition includes their score diagrams, followed by the proofs and references.

Sound an A at 440 Hz and ask for its seventh harmonic. The answer is 3080 Hz. Bring it down two octaves and it becomes 770 Hz. On a piano the nearby key is G. If the A moves, which G should move with it, and by how much?

The score I propose begins with the instruction that produced the sound. Write “seven quarters of this A” and the seventh has a musical identity before anyone chooses a concert pitch. The same idea lets a composer ask for a tempered interval, borrow the entrance of another voice, or pick up an entire cadence and attach it to a new note. The way into the language is through the sound of a string.

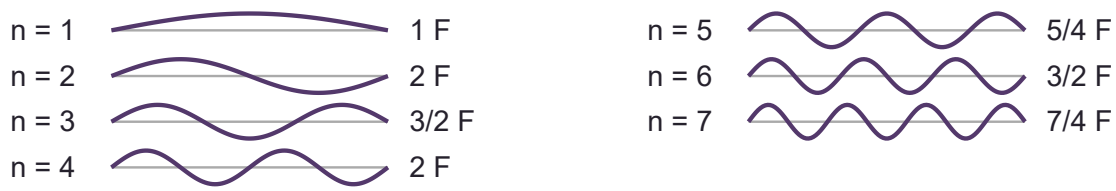
Imagine that Beethoven could put a tuning instruction beside a violinist’s G. Does he want it to belong to the A7 chord beneath it, or to take its pitch from the F♯ toward which the line is moving? This is a hypothetical passage. A violinist can hear and argue for either reading. A composer who has made the choice can write it down and give the player that knowledge. This is the sense in which the composer gains full control over the intended pitches, one note at a time.

Section 1 A brief history of the 12-note system

One string and several notes

Hold a string at both ends. In its simplest motion the whole length bows one way, then the other. The next mode has a stationary point in the middle. The next has two interior stationary points. For an ideal flexible string of length L, the wavelength of mode n is $2L/n$. With wave speed held constant its frequency is nF, where F is the fundamental. A plucked string may excite several modes at once. Their relative strength depends, among other things, on where the string was plucked. Wolfe, String harmonics.

Bring the third harmonic down an octave and you have 3/2 of the fundamental, a pure fifth. Bring the fifth and seventh harmonics down two octaves and you have 5/4 and 7/4. With the fundamental these give the four pitches of a harmonic seventh chord. You can see this relationship most clearly in the arithmetic by leaving the four partials in their natural register. They stand next to one another as 4, 5, 6 and 7 times the same lower frequency.



Standing waves on one ideal string. At right of each mode is its frequency folded into the octave from F to 2F. Octaves appear at the upper boundary.

Why twelve fifths nearly meet seven octaves

A player tuning by ear can take a fifth, then another fifth from the note just found. Each move multiplies the starting frequency by 3/2. Moving an octave multiplies it by 2. After twelve fifths the result is close to seven octaves. Divide the first result by the second and the small excess is exactly 531441/524288, about 23.46 cents. A cent is one hundredth of an equal-tempered semitone. This excess is the Pythagorean comma. Duffin on tuning and commas.

The calculator lets you try the question with five, twelve or even 665 fifths. Better near meetings exist. An exact meeting would require a positive power of 3 to equal a power of 2. That is impossible. A keyboard with twelve pitch classes is one practical answer to this old tuning problem. Unequal temperaments distribute the discrepancy unevenly. Twelve-tone equal temperament makes every semitone the same ratio, $2^{1/12}$. Its fifth is seven of those steps and twelve such fifths span exactly seven octaves. The pure fifth and the tempered fifth express different intentions. Proof 3.

In 1563 the mathematician Giovanni Battista Benedetti sent the composer Cipriano de Rore a little musical puzzle. Sing the indicated pure intervals through two measures, then repeat them. The singers reach the same written notes a syntonic comma, 81/80, higher on each pass. The letters appeared in print in 1585. Duffin reproduces the music, which puts the difficulty in a form one can sing. Duffin, paragraphs 20–21 and example 1a.

Which G belongs to A?

Here are three readings of the keys A, C♯, E and G. The first fixes positions in twelve equal steps. The second takes four consecutive harmonic partials. The third reaches the pitches through octaves and pure fifths. Read down the G column before listening to the cadence in Section 2.

A seventh chord above 440 Hz. Decimal values rounded to three places.

Tuning	A	C♯	E	G
12 equal steps	440	554.365	659.255	783.991
Partials 4–7	440	550	660	770
Pure fifths and octaves	440	556.875	660	782.222

For the third reading, four upward fifths reduced by two octaves give the third 81/64. Two descending fifths, each moved up an octave, give the seventh 16/9. The Pythagorean third, 81/64, exceeds the harmonic third, 5/4, by a factor of 81/80, the same syntonic comma that lifted Benedetti's singers. A violinist can choose either on the same string. If a composer wants a performer or a program to repeat the choice, the written G or C♯ needs more information. The fractions we used to explain the difference can themselves become the score.

Section 2 The Relative Music Theory

Write what one note takes from another

Suppose a note should sound a pure fifth above a particular voice, enter with a second voice, and last half as long as that second voice. These are three decisions. A change in pitch in the first voice should travel to the new note. A change in timing in the second voice should travel there too. Writing the

resulting frequency and two times gives a faithful account of one performance. It leaves a later reader to guess which changes ought to travel.

In Relative Music Theory (RMT), each field can carry its own expression. Here is one note as it might appear in the editor's saved score. The bracketed numbers identify other events. The three letters `f`, `t` and `d` read their frequency, start time and duration respectively.

```
{
  "id": 9,
  "frequency": "(3/2) * [3].f",
  "startTime": "[8].t",
  "duration": "[8].d / 2"
}
```

With event 3 at 440 Hz, event 9 sounds at 660 Hz. Move event 3 to 432 Hz and event 9 follows to 648 Hz. Move event 8 later and event 9 enters later at the same pitch. The event supplying a frequency can sound later in the piece. A composer can tune a leading voice to the chord it is approaching. The evaluator needs the referenced value before it can calculate the dependent value. The listener can hear them in the opposite order.

What the relationship contributes

Musicians already have several ways to say "a fifth above." Lou Harrison's free-style just intonation, described by Larry Polansky, builds the tuning as the music proceeds, one chosen ratio after another. In the Siren software a relative pitch can store a ratio and a reference pitch. Both anticipate the pitch operation above. Hierarchical scores have also allowed a composer to place a musical structure at a new pitch and time. Rubato represents musical events relative to anchors, and score spreadsheets make dependencies between cells explicit. Polansky · Pope · Buxton and colleagues · Thalmann · Nash.

The proposal made on cybercyril.com is to use the choice of a source *for each property of an event* as ordinary score material and to let a group of such events be played again from a different anchor. A note can take its pitch from one voice, its entrance from another and its length from a third. Version 2 of RMT Compose, the browser editor built for this notation, gives that proposal an editable file format and arithmetic that distinguishes an exact construction from an estimate. This is a claim about the particular language and its implementation. One can spell the same interval in cents, ratios or another reversible notation. What has to survive a change of reference is the interval information. What has to survive an independent edit is the composer's choice of dependency. Proofs 1 and 2 · Dated web article.

Write the chord once

Start a major chord with a root, its 5/4 and its 3/2. Collect the three events together and expose the pitch they all take as their reference. At A = 440 Hz the chord is 440, 550, 660 Hz. Attach it to D = 880/3 Hz and the same instructions give 880/3, 1100/3, 440 Hz. A composer may shift one voice by an octave to change the voicing. The saved group can be a chord, a scale, the subject of a fugue or a whole passage.

In RMT Compose this group is a *module*. Dropping one on a pitched event makes that event the source of the module's pitches and places its entrance at the event's start or end, as chosen. Durations written against `base.d`, the score's beat, stay in beats instead of stretching to the target. A long target

note can therefore receive a short motif at its usual speed. Attack, release and amplitude belong to the rendering of an event. They need not enter the definition of a harmonic interval. RMT Compose v2, source and example modules.

Listen to a dominant seventh resolve

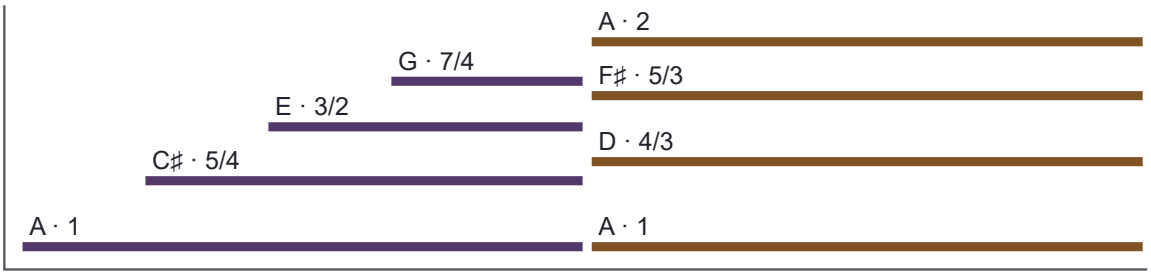
Draw time from left to right and frequency on a logarithmic vertical axis. Every octave occupies the same height, and a given interval spans the same distance wherever it starts. In the example, A enters first, then its third, fifth and seventh. All four give way to a D-major chord. The diagram below comes from the original article and shows which pitches continue or change. Read A7 here as *A dominant seventh*, with G as its seventh.

Listen first to the two single cadences at the same base frequency. In the harmonic version the successive entrances give the proportions 4:5:6:7. The arriving D is 4/3 of the initial A, and its major third is 5/4 of that D. In the equal-tempered version the steps are selected from twelve equal divisions. Then try the three-cadence versions. Each arrival supplies the next departure. Changing the initial frequency moves the whole experiment. The sine waves and staggered entrances are a way to compare the tunings by ear. They leave the choice of voicing and timbre open.

What should the violinist do with the seventh?

Take the descending violin line A–G–F♯, beginning on the upper A at 880 Hz. Give the underlying A a frequency of 440 Hz and let the arriving F♯ be 5/3 of that A, or 2200/3 Hz. If the composer hears G as the chordal seventh of A, its instruction is `(7/4) * [1].f`, where event 1 is the supporting A at 440 Hz. The G is exactly 770 Hz and descends to F♯ by a ratio of 21/20. If G is a passing note whose job is to approach F♯ by the particular semitone 16/15, the instruction becomes `(16/15) * [3].f`, where event 3 is the coming F♯. This G is 7040/9 Hz, about 782.22 Hz. The written violin part could show the same three note names in both cases.

Compare the two endings in the violin example. Move the destination F♯ while holding A steady. The G belonging to A stays put. The G belonging to the approaching F♯ moves with it. Neither the word “seventh” nor the word “passing” dictates either ratio. It is the composer who specifies the pitch consequence of hearing the note that way. The violinist can still shape the descent with bow, timing and vibrato. One precise instruction has passed from the composer to the player without requiring the player to reconstruct it by intuition.



A7 resolving to D. Each line gives one voice as a ratio to the initial A. Time runs rightward. Vertical distance measures log frequency.

Section 2.b

Scope of application and examples

1. From the diagram to an instrument for composing

The original article asked for a graphical editor in which one could drag these groups onto a note. RMT Compose now offers that experiment. Inspect an event and its expressions, reattach a chord, then listen. Score files retain the expressions as text. Internally the editor compiles them into instructions and follows dependencies between individual fields. A circular definition cannot supply its own starting value. For performance, the resulting frequencies and times are converted to numbers the audio system can use. Open RMT Compose · Version examined here.

The distinction between an exact instruction and a close reading matters. The harmonic seventh 7/4 needs only two integers. The editor keeps rational numbers as fractions with BigInt numerators and denominators. A twelfth of an octave as an interval step needs a different expression, `2^(1/12)`. Keeping the power allows twelve such steps to return exactly to a factor of two. The same rule works with seven equal divisions of 7/4, thirteen of 3, or seven of the compound period $1 + \sqrt{2}$. An explicitly approximate number, written for example `~1.4983`, carries that status through arithmetic. The finite evaluator also falls back to approximation outside its supported symbolic operations and resource limits. Exactness here concerns the stored construction. A loudspeaker operates at finite precision. Proof 4 · Twelve-step test · Compound-period test.

2. Transcribe a musical decision

When transcribing a performance, hearing two frequencies gives their measured ratio. It does not settle which voice the performer treated as the reference. A transcription can mark a measured value as approximate, or record a relationship supported by the player, the instrument or the score. These readings answer different questions. A tradition's tuning and phrasing need the knowledge of its musicians. The editor offers a place to write what is learned from them.

The original site illustrates reuse with a module for Bach's ascending canon from *The Musical Offering*. Follow a return of the subject and ask which earlier or later event should supply its pitch. Another voice may supply its entrance. A cadence can lean toward a future bass even though that bass has yet to sound. The choice is available to the arranger and has audible consequences. Other scores have foregrounded interval instructions, and adaptive instruments can select tuning from performance context. The choice here can be fixed explicitly in the score. Inspect the canon module · Canon sources · Zhu and Sapp · Trueman and colleagues.

The two MIDI scores in this edition are a four-part setting of Gruber's *Silent Night* and Bach's Prelude in C major, BWV 846. Each has a file in twelve-tone equal temperament and two RMT versions. The RMT versions tune pitches in relation to the tonic and the chord roots. In these exports each note uses its nearest MIDI key and a channel of its own while it sounds. With the pitch-bend range set to ±2 semitones, rounding changes the intended pitch by less than 0.0123 cent. The comparison page plays the plain file and the selected RMT version as sine tones with identical notes, onsets, durations and MIDI velocities. Export MIDI source.

3. Explore the space between keys

Take an augmented chord, such as C–E–G♯. If each of its thirds is a pure 5/4, the pitches above a root of frequency r are r, $5r/4$ and $25r/16$. Continue for one more third and the result is $125r/64$, about 41.06 cents below the octave. If the chord divides an octave into three equal parts, its step is $2^{1/3}$, and

the third step gives exactly $2r$. Both constructions can appear on the same three piano keys. A score can now keep the difference visible.

In the listening examples the period itself changes. A chord spanning a factor of three places 1, 5/3, 7/3 and 3 above its root. A later example plays as one chord the six pitches that bound five equal steps inside a factor of five. Each step has ratio $5^{1/5}$, and the outer pitches form an exact 5:1. In the two scales, try listening for the return at the boundary of a period. In the repeated V–I, by contrast, the bass is folded back into range while the chain of new roots continues. An octave shift keeps the example audible without cancelling its history of fifths. Proofs 3 and 4.

Listening guide

1. **An open-ended chain of cadences.** Each cadence takes the preceding resolution as its new root and brings that root back into the starting octave.
2. **A chord spanning a 3:1 period.** Hear 1, 5/3, 7/3 and 3 resolve to 1, 4/3, 5/3 and 2.
3. **From a descending chord to a new voicing.** Follow 1, 7/9, 5/9, 3/9 into a chord using 3/5, 4/5, 1 and 6/5.
4. **A second open-ended progression.** A minor sonority moves through major and seventh sonorities before the next cycle.
5. **The nine-note "Tesla" scale.** Over a 3:1 period, hear 1, 11/9, 13/9, 5/3, 17/9, 19/9, 7/3, 23/9 and 25/9, followed by a chord. The name is the author's.
6. **Ten pitches within a 5:1 period.** Odd numerators from 5 to 23 over a denominator of 5 give the scale. The concluding chord reaches the period.
7. **Five equal divisions of 5:1.** The six pitches with factors $5^{k/5}$ for k = 0, …, 5 sound together.
8. **Kids' Games improviser.** Cyril Monkewitz's improvised arrangement draws on Gérard Massini's Kids' Games, Fanfare. Voices enter from a small library of themes as a randomized procedure chooses repetitions, transitions and modulations.

4. Build a library of relationships

A useful library entry would contain more than a chord name. An augmented chord made with two 5/4 steps belongs beside one made by dividing the octave in three. Each entry can carry its equations, voicing and a sound example. A composer could tag the situations in which an entry has proved useful. Different listeners may describe the same resolution quite differently. The library can preserve these accounts alongside an interval recipe that anyone can inspect and replay.

5. Let one phrase invite another

The improviser based on Gérard Massini's *Kids' Games, Fanfare* draws on a small collection of phrases and makes choices among them. It returns the question of a reference to the scale of a whole phrase. Once a phrase knows how to attach to an arrival, a program can choose its successor and still leave their musical connection audible. The linked standalone instrument adds tempo control. Hear the improviser.

How far can such a language reach? The ideal harmonic series already supplies infinitely many distinct pitches within one octave after octave reduction. From a given reference, a fraction names any positive rational multiple exactly. Fractions can also come as close as desired to every positive frequency. Symbolic powers name more intervals exactly, including the equal-tempered semitone. There are still more real numbers than finite expressions. The freedom this gives a composer is quite concrete: choose a relationship without asking for a spare key, keep it as precisely as the score admits, and let another note depend on it. Proofs 3 and 5.

About this edition

I dedicate this work to my son Zénon Yota.

Cyril Monkewitz wrote the original article and conceived the notation and musical examples. The article was publicly archived by November 2023.

The improviser uses Gérard Massini's commissioned composition *Kids' Games, Fanfare*. Massini retains the right to reuse his composition.

Mathematical proofs

What must a score remember if a musician can move any note and expect the intervals written from it to follow? We model each pitched event by a positive frequency in hertz. A transposition multiplies every frequency by the same positive number. These assumptions concern tuning. Rhythm, timbre and the perception of pitch involve further questions.

1. What survives a change of reference?

Proposition. The frequency ratios to one chosen note determine an ordered collection of pitches up to a common transposition.

Proof. Choose the first note, of frequency f_0. For each note write $r_i = f_i / f_0$. Transposing by a factor s multiplies both parts of every ratio by s, so every ratio stays the same. Conversely, suppose a second collection has the same ratios and its first note has frequency g_0. Then each frequency in it is $g_0 \times r_i$. A single transposition by g_0 / f_0 takes the entire first collection to the second.

It follows that every description which is unchanged by transposition and distinguishes any two collections that are not transpositions of each other must distinguish precisely these ratio patterns. Any one-to-one encoding will do. For example, an exact interval in cents c determines its ratio by $2^{c/1200}$. The necessity concerns the information a score carries. Lewin's generalized interval systems give an earlier, broader treatment of intervals and transpositions. Lewin, *Generalized Musical Intervals and Transformations*, chapter 2.

2. Which note should follow an edit?

Proposition. Even exact frequencies for an entire chord can leave its intended response to an edit undetermined.

Proof by example. Sound a bass at 440 Hz, a middle note at 550 Hz and a top note at 660 Hz. Write the top as a fifth of the bass and its rule is 440 × 3/2. Write it as a minor third of the middle note and its rule is 550 × 6/5. Both give 660 Hz under the original tuning and agree after every common transposition. Now lower the middle note to 540 Hz and hold the bass. The first rule still gives 660 Hz. The second gives 648 Hz. Since the starting frequencies are identical, they alone cannot distinguish the two instructions.

A score that supports both readings must preserve enough information to choose between them. In RMT the composer makes the choice by writing the source note and the interval into the expression for the top note.

3. How many pure fifths fit into an octave?

Proposition. No finite collection of pitch classes contains every successive pure fifth, even when frequencies an octave apart count as the same class.

Proof. If a class recurred, some positive number k of fifths would span a whole number m of octaves. This would give $(3/2)^k = 2^m$, hence $3^k = 2^{m+k}$. The left side has a prime factor of 3. The right side is a power of 2. Therefore the classes never repeat. Twelve fifths, for example, exceed seven octaves by the ratio 531441/524288, about 23.46 cents.

The ideal harmonic series gives an even more direct example. Above a fundamental F, two different odd-numbered partials always belong to different octave classes. An upward shift of one or more octaves makes an odd partial number even, so it cannot produce another odd partial number. Infinitely many odd partial numbers therefore give infinitely many exact classes.

4. When does a divided period close exactly?

Proposition. For any positive period ratio P and any positive integer n, exactly one positive step ratio produces P in n equal steps.

Proof. A repeated step x gives the product x^n. Positive numbers have unique positive nth roots, so the required step is $P^{1/n}$. More generally, multiplying steps with the same base adds their exponents. Steps P^a and P^b span P^{a+b}. Any finite sequence of rational exponents summing to one closes on P, including sequences with unequal or descending steps.

Twelve steps of $2^{1/12}$ reach the octave. Seven steps of $(7/4)^{1/7}$ carry 440 Hz to exactly 770 Hz. Each step subdivides the natural seventh, whose own ratio is simply 7/4. The same proof applies to a compound period such as $1 + \sqrt{2}$. Replacing an irrational step with a rounded decimal changes its product and loses exact closure.

5. How closely can a score specify a frequency?

Proposition. From an exact positive reference F, integer ratios reach every positive rational multiple of F exactly. They also reach within any chosen positive error of every positive frequency.

Proof. A positive rational multiple has the form $F \times p/q$ for positive integers p and q, which gives its expression immediately. For any positive target x and positive tolerance ε in hertz, choose a denominator q so large that the grid spacing F/q is smaller than both x and 2ε. Round qx/F to the nearest integer p. It is positive. Its grid point $F \times p/q$ lies at most half a grid spacing from the target, hence within ε.

Exact symbolic powers name further values, including $\sqrt{2}$. There is also a limit shared by every language of finite scores over a finite alphabet. List its strings by length and then alphabetically within each length. This enumerates all its expressions, so at a fixed reference they can name at most countably many exact values. The positive reals are uncountable. Arbitrary accuracy is attainable for every target in this model, while exact naming depends on the target and the operations available in the language.

Sources and precedents

1. University of New South Wales, Music Acoustics. *Strings, standing waves and harmonics*. The diagrams show how one ideal string supports modes at integer multiples of its fundamental. The discussion of real strings explains why measured partials can depart from these values. Read the illustrated account.

2. Ross W. Duffin (2006). “Just Intonation in Renaissance Theory and Practice.” *Music Theory Online* 12(3). In paragraphs 20–21 and example 1a, Duffin reconstructs the first of Benedetti’s tuning puzzles. Successive pure intervals bring singers back to a familiar written note at a different frequency. Example 1b sets the passage in meantone temperaments, one way to avoid the drift. Read the analysis and see the musical examples.

3. Larry Polansky (1987). “Paratactical Tuning: An Agenda for the Use of Computers in Experimental Intonation.” *Computer Music Journal* 11(1), 61–68. Pages 64–65 describe Lou Harrison’s free style, in which a succession of chosen ratios builds the tuning as the music proceeds. This is a direct precedent for choosing a local pitch reference. Journal record · Read the discussion.

4. William Buxton, William Reeves, Ronald Baecker and Leslie Mezei (1978). “The Use of Hierarchy and Instance in a Data Structure for Computer Music.” *Computer Music Journal* 2(4), 10–20. A composer can store a passage once and reuse it with changes of pitch, time or dynamics. Sections 1.2.3 and 4.1.2.2 describe the instance mechanism. Read the authors’ paper.

5. David Lewin (2007; first published 1987). *Generalized Musical Intervals and Transformations*. Oxford University Press. Chapter 2 defines generalized interval systems. It places the transposition argument in Proof 1 within a much broader account of musical intervals. Publisher’s edition and contents.

6. Stephen Travis Pope (2007). *The Siren 7.5 Reference Manual*. The RatioPitch and RatioMagnitude classes give a particularly close precedent. A pitch can carry a ratio, and the implementation evaluates it against its reference pitch. Consult the class reference · Inspect the later implementation.

7. Florian Thalmann (2007). *Musical Composition with Grid Diagrams of Transformations*. Master’s thesis, University of Bern. Pages 18–19 describe Rubato Composer’s hierarchical MacroScore, including satellite notes whose coordinates are relative to an anchor. Read the account of satellite notes.

8. Chris Nash (2014). “Manhattan: End-User Programming for Music.” *Proceedings of the International Conference on New Interfaces for Musical Expression*, 221–226. Manhattan puts spreadsheet formulas in a sequencer. A cell can read another cell and respond when its value changes. Read the paper and score examples.

9. Dan Trueman, Aatish Bhatia, Michael Mulshine and Theo Trevisan (2019). “Tuning Playfully: Composed and Adaptive Tunings in bitKlavier.” *Computer Music Journal* 43(2–3), 67–88. The authors show how the choice of tuning reference can depend on what the performer plays. In RMT, the composer can specify the reference of a particular scored event. Journal record · Read the paper.

10. Julie Zhu and Craig Stuart Sapp (2021). “Ratioscore: A Text-Based System for Just Intonation.” *Proceedings of TENOR ’20/21*, 205–211. Sections 2.1–2.2 describe the reference pitch of a voice and the use of exponents to write its ratios. Their quartet illustrates the use of exact ratios in an executable score. Read the paper.

11. Andrea Agostini and Daniele Ghisi (2025). “Just Intonation in bach.” *Proceedings of TENOR ’25*, 241–252. This later design combines a tempered spelling with a just-intonation component and represents rational factors by prime exponents. It gives another way to see why the arithmetic of an interval and its spelling on the page are separate choices. Read the paper.

12. Markus Lepper and Baltasar Trancón y Widemann (2026). “F-operator: A Proposal for a Generative Notation for Frequency Factors in Music.” *Proceedings of TENOR ’26*. The authors give a common notation for frequency factors in different tuning systems. Their rational exponents also express equal divisions. Read the paper.

13. Cyril Monkewitz (2023). *Relative Music Theory*. Section 2 gives a note with separate references for frequency, start, length and amplitude, then describes attaching a chord or theme to a source note. The Internet Archive capture establishes that this text was public by 1 November 2023. Consult the archived original.

14. Cyril Monkewitz (2024). *Bach’s Neverending Canon*. Playable RMT arrangement of J. S. Bach’s *Canon a 2, per tonos* from *Musical Offering*, BWV 1079 (1747). The whole-tone tuning used in the demonstration is Monkewitz’s choice. Read the arranger’s account · See the player captured 13 November 2024 · Inspect the arrangement and player · Hear the Netherlands Bach Society’s performance.

15. Cyril Monkewitz (2025–2026). *RMT Compose*. The February 2025 capture records the publicly accessible first-generation editor. The source revision examined here includes exact fractional steps across compound tuning periods. See the editor captured 14 February 2025 · Inspect the v2 source.